Calculations of SCR, Technical Provisions, Market Value Margin (MVM) using the Cost of Capital method, for a one-year risk horizon, based on an ICRFS-Plus™ single composite model and associated forecast scenarios for the aggregate of multiple LOBs

Introduction

There are many interpretations of the Solvency II Capital Requirements (SCR) and Market Value Margins (MVM) in the context of long tail liabilities. In particular, there are several CEIOPS' Quantitative Impact Studies (QIS) and there are so-called *Internal* models. Furthermore, as with a Balance Sheet, there are two sides in the Solvency II regulation, liabilities and assets.

The approach presented here can be categorised as an *Internal* model, focusing on liabilities. Here the *Internal* model is based on a single composite model for all long tail liability LOBs.

From a composite model for multiple LOBs all necessary details can be deduced - mean loss, risk capital for a particular quantile (based on VaR/T-VaR), and MVM using the Cost of Capital method.

In the context of this memo, the following definitions hold,

BEL – best estimate liability, deduced from the mean loss as a present value of mean loss,

RC – risk capital to cover non-hedgeable (that is, insurance) risk; defined from VaR or T-VaR for a particular quantile (e.g. 99.5%) of the predictive aggregate loss distribution; defined for each future calendar period until run-off,

SCR - Solvency Capital Requirement - the risk capital required for the first year; RC(1).

MVM(*k*) – present value cost of holding risk capital RC; defined for each future calendar year k until run-off,

MVM (or Total MVM) - the sum of MVM for all run-off periods,

Insurance liabilities fair value - the sum of BEL and total MVM. This quantity is also referred as **Technical Provision (TP).**

TP = BEL + MVM

In each year k we have $TP(k) = BEL(k) + MVM(k)$

The aggregate TP is the sum of the individual year TPs, including Present Value discounting, since these sums will only be required at the start of their respective years.

For the purpose of Insurance Liabilities fair value calculation and Solvency II we assume,

- Insurer does not borrow the capital, it raises it instead. That is, SCR is raised at inception (evaluation date), $RC(k)$ for $k > 1$ is raised in year k.

- In one case out of 200 (99.5%) insurer is ruined.
- The liabilities fair value is relevant to the evaluation date. This quantity is assumed to be funded from premiums; the total MVM is estimated at inception (evaluation date).

RC and the associated MVM can be computed for **one year risk horizon** and **ultimate risk horizon**. The Solvency II regime favours the former.

In order to compute MVM using the Cost of Capital approach and SCR for both one year risk horizon and ultimate risk horizon for the aggregate of all long tail LOBs and each LOB the following critical information is required:

- Probability distributions of paid losses (liability stream) by calendar year (k=1,..,n) and their correlations, for each LOB and the aggregate of all LOBs.

- Probability distributions of total reserves for each LOB and the aggregate of all LOBs.

- Probability distributions of the aggregate paid losses from calendar year k to calendar year n for each LOB and the aggregate of all LOBs. This is required for each k raging from 1 to n, where complete run-off is achieved at the ultimate calendar year n.

- The above mentioned distributions enable the calculation of, amongst other statistics, the $\mathrm{VaR}_q(k)$ for the paid losses (total loss) in calendar year k, at percentile q, for each LOB and for the aggregate of all LOBs.

For a discussion of the ultimate risk horizon calculations see
http://www.insureware.com/Library/SolvencyII/solvencyii_ultimate.php

One year risk horizon

With the one year risk horizon, the risk capital fund is raised at the beginning of each year and is released at the end of each year. At the conclusion of each year the unused portion of the fund is returned to the capital provider together with the interest earned at the risk free rate, and the MVM, the risk premium paid for access to the fund.

If we temporarily ignore the correlations between the loss distributions for the future calendar years, the amount of the risk capital $\mathrm{RC}(k)$ raised at the beginning of year k is $\mathrm{VaR}_{99.5\%}(k)$. The $\mathrm{VaR}_{99.5\%}(k)$ is a quantity calculable from the distribution of losses for calendar year k, these distributions are available to us "now" at the beginning of year 1. We refer to these as the unconditional distributions.

Assuming the spread above the risk free rate is s, and the risk free rate is d, the MVM for year k is given by the Cost of Capital method as

(1) $\mathrm{MVM}(k) = \mathrm{VaR}_{99.5\%}(k)\frac{s}{(1+d)^k}$, where s = spread.

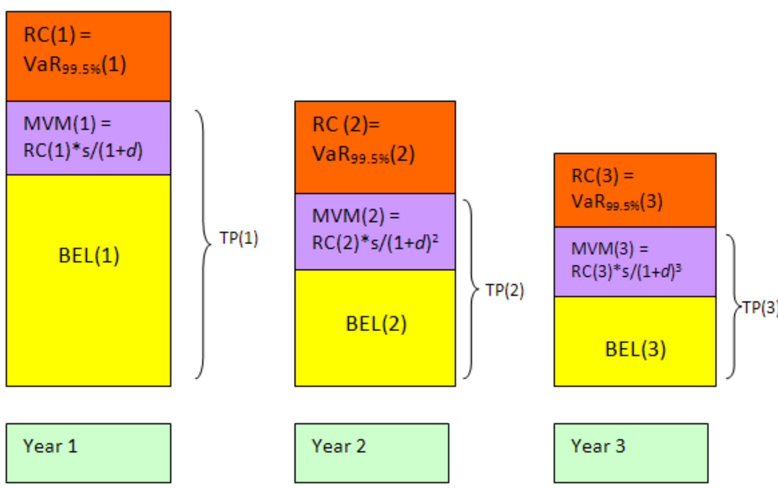
In this formula the factor

$$\frac{s}{(1+d)^k}$$

represents the *spread*present value factor*. The MVM is paid to the risk capital provider at the conclusion of the year and so MVM for year k can be considered at accumulate interest at the risk-free rate for k years. Present value adjustment for the BEL differs from this by half a year. Payments of losses occur roughly evenly throughout the year and so we treat them for discounting purposes as if they all occur exactly half way through the year. The present value factor for BEL is accordingly

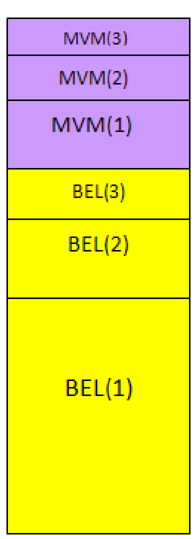
$$\frac{1}{(1+d)^{k-1/2}}.$$

In practice a more detailed analysis may treat the d as a random variable conditional on k, in this case the *present value factor* could be a mean value computed by simulation from the relevant rate models.



Recall, the Technical Provision (TP), the sum the insurer must hold, is the sum of BEL and MVM.

(2) Total MVM = $\sum_{k=1}^{n} \mathrm{MVM}(k)$



The aggregate TP is what needs to be shown on the current balance sheet to demonstrate that the company can meet its obligations in each future calendar year, including the cost (the total of the MVMs) of maintaining an adequate risk capital fund to guard against severe loss over-runs.

At the conclusion of calendar year k the risk capital fund RC(k) is returned to the capital provider minus an amount equal to the excess of actual loss over the expected or best estimate of the loss, if this is positive, in other words, minus the drawings from the risk fund.

In addition the capital provider always receives MVM(k) plus the risk free rate on RC(k) at the end of year k. If the loss is below the expected value then the difference is initially retained by the insurer, and may even be released as profit.

Each RC(k) k=1,...,n is raised from the capital providers, whereas the MVM(k) is funded by the policyholders.

Bringing in Correlations between Future Calendar Years

If we now factor in the correlations between the loss distributions for the future calendar years we will see that the change in the above is only in formula (1) in the case k=1.

When we are considering the situation that arises after the first year is in distress we must naturally consider the possibility that our models for future years will need to change. Although it may be argued that the original model is still correct and the extreme result was simply a result of process variation already factored into the model, this would seem unlikely if the 95th percentile or above had been reached. In this case it is more prudent to update the model especially in respect of calendar year (or inflation) parameters. In practice a further decision would be required at this point since both parameter changes *and* process variability are likely to have contributed to the result and so the proportional contribution of each would need to be estimated.

If the model correctly incorporates parameter uncertainty as well as process variation then the formation of conditional distributions for outstanding future years is achieved as the model is re-estimated with the first future year data. Such a model includes the correlations between the distributions for future calendar years, so once we know the actual outcome in the first year it enables us to recompute the distributions for the following years consistent with that result.

In forming the risk fund at the start of the first year we do not assume that the coming year will be an exceptionally bad year, we simply ask, whether, if it is exceptionally bad, the risk fund will enable us at the end of the year to meet the extra obligations and to restore confidence in provisions for later years (years 2+). In such a contingency the provisions that were originally allocated for years 2+ will seem insufficient since the re-estimated models will have forced significant upgrading of loss expectations. The risk fund will be adequate if it covers not only the excess losses in year 1 but also the cost of the upgrading of the provisions for years 2+.

The costs of these upgrades can be estimated precisely by the use of the conditional distributions. The extra money that will be needed in year k is equal to the difference between the technical provision as calculated based on the conditional distribution and the technical provision based on the unconditional distribution.

Let ξ represent "Year 1 in distress at the 99.5 percentile".

$$\text{Upgrade-to-provision-for-year } k = \quad \mathrm{TP}(k \mid \xi) - \mathrm{TP}(k),$$

we refer to this quantity as $\Delta TP(k)$.

Now, $TP(k) = BEL(k) + MVM(k)$,

and both of these quantities are computed from the distribution of the losses in year k, $\mathbf{L}_k$.

Example: two year run-off after first year is in distress

In a simple example where the run-off period is two years, and all risk capital is invested and assumed to get (at a minimum) the risk free rate, the formulas are very straightforward and can be summarised as follows:

$MVM(2) = (VaR_{99.5}(2) * s) * PV(2)$

$\Delta VaR_{99.5}(2) = VaR_{99.5}(2|\xi) - VaR_{99.5}(2)$

$\Delta MVM(2) = \Delta VaR_{99.5}(2) * s * PV(1)$

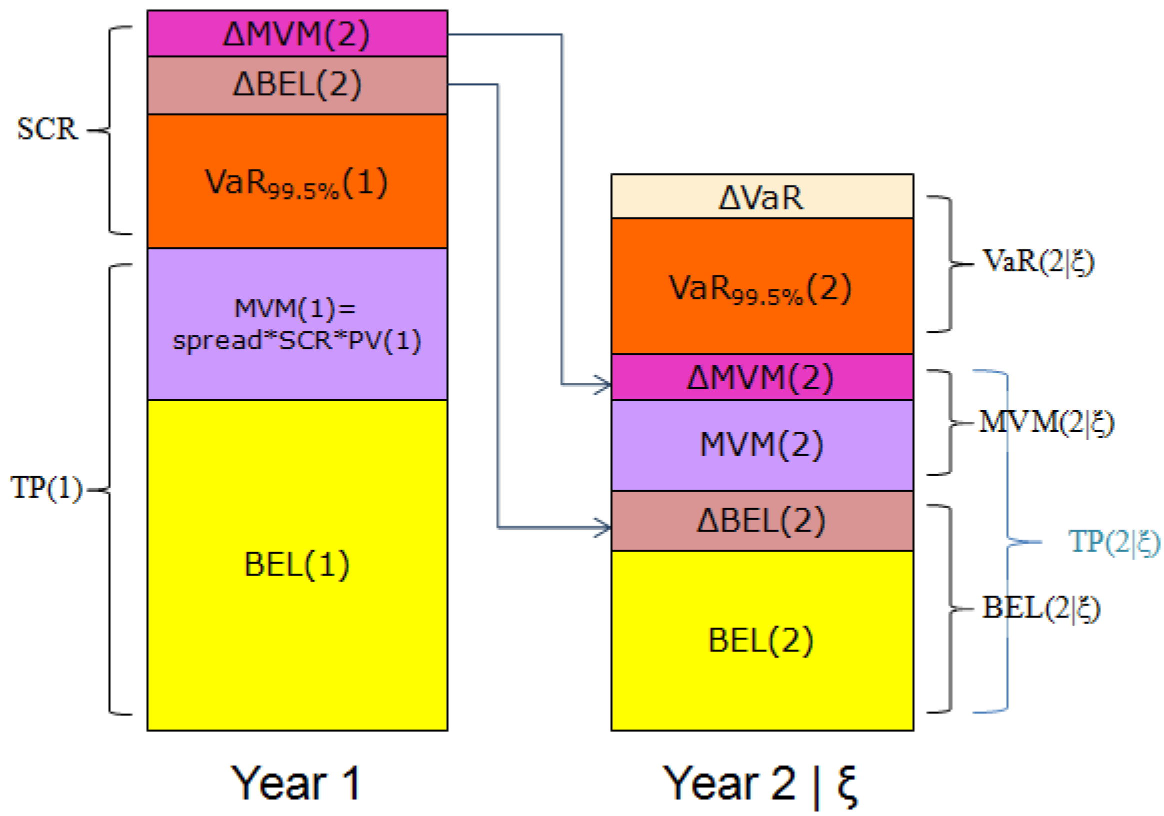
$BEL(2) = E[L_2]*PV(1.5)$

$\Delta BEL(2) = (E[L_2|\xi] - E[L_2]) * PV(0.5)$

$\Delta TP(2) = \Delta BEL(2) + \Delta MVM(2)$

$MVM(1) = s * SCR * PV(1)$; where $SCR = VaR_{99.5}(1) + \Delta TP(2)$

$MVM = MVM(1) + MVM(2)$



Now, if the first year is in distress then:

$VaR_{99.5}(1)$ is consumed.

MVM(1) grows to SCR * s which is returned to the capital providers along with the risk free rate, d, on the total risk capital. Total amount returned is: SCR(d+s)

ΔBEL(2) is used to adjust BEL(2) at the beginning of year 2; $E[L_2|\xi]$ * PV(0.5) is available at the beginning of year 2.

ΔMVM(2) is used to adjust MVM(2) at the beginning of year 2; MVM(2) / PV(1) + $\Delta VaR_{99.5}(2)$ * s * PV(1) is available at the beginning of year 2.

Any return on ΔBEL(2) and ΔMVM(2) in the first year is returned to the capital providers as part of the risk free rate calculated on the SCR which is why the PV adjustment is one year less than for the MVMs and BELs respectively.

If the risk capital is assumed not to be invested, then the equations change so that MVM is higher as risk free return is added to it, MVM(2) = ($\text{VaR}_{99.5}(2)$ * (s + d)) * PV(2)

ΔMVM(2) = $\Delta\text{VaR}_{99.5}(2)$ * (s + d) * PV(1)

MVM(1) = (s + d) * SCR * PV(1); where SCR = $\text{VaR}_{99.5}(1)$ + ΔTP(2)

MVM = MVM(1) + MVM(2)

Unless specified otherwise, we assume that risk capital including VaR component earns interest at the risk-free rate.

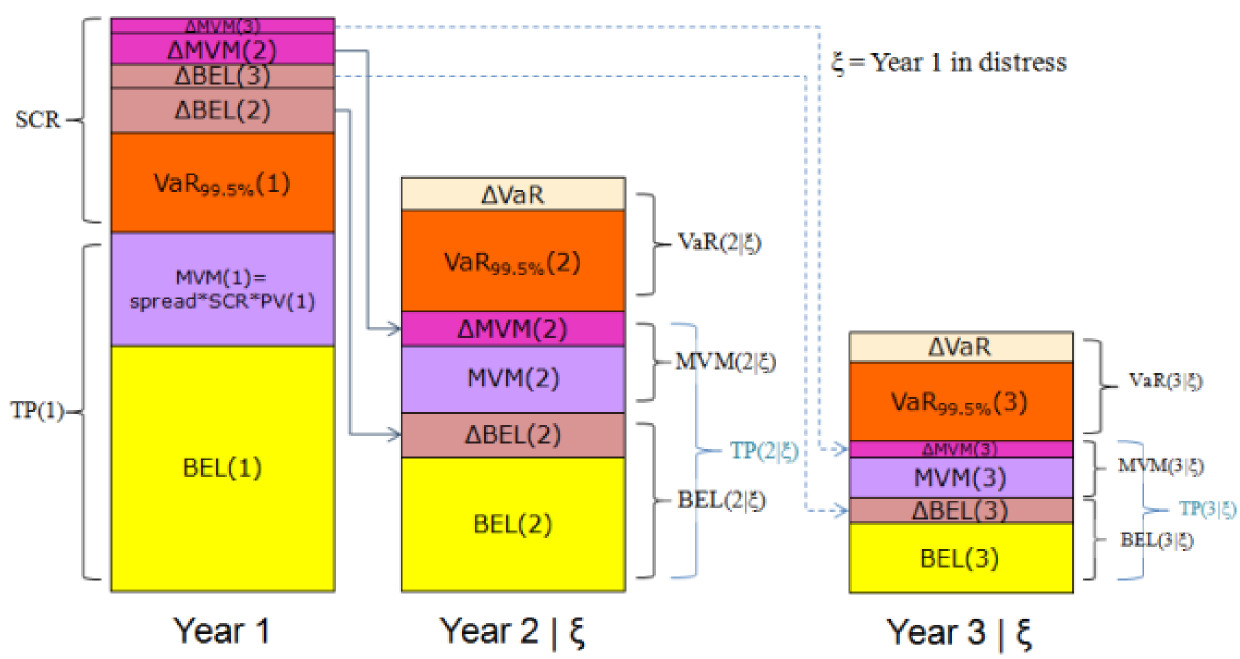
General run-off formulation

Generally speaking,

$$\text{BEL}(k) = \frac{\text{E}(\mathbf{L}_k)}{(1+d)^{k-1/2}} \text{ and } \text{MVM}(k) = \text{VaR}_{99.5\%}(\,\mathbf{L}_k)\frac{s}{(1+d)^k}, \text{ so}$$

$$\Delta\text{TP}(k) = \frac{\text{E}(\mathbf{L}_k|\,\xi) \;-\text{E}(\mathbf{L}_k)}{(1+d)^{k-1.5}} + \frac{s*(\text{VaR}_{99.5\%}(\,\mathbf{L}_k|\,\xi) - \text{VaR}_{99.5\%}(\,\mathbf{L}_k))}{(1+d)^{k-1}},$$

where s = spread.



So the total upgrade of the provisions for years 2+, given that year 1 is in distress at the 99.5th percentile is,

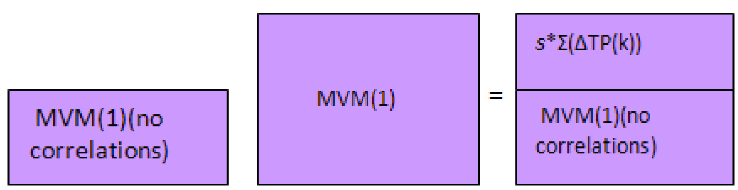
$$\Delta\text{TP} = \sum_{k=2}^{n} \Delta\text{TP}(k)\ .$$

This amount that must be available from the risk fund at the end of year 1 so that the health of the reserves can be re-established after a highly distressed year. As far as the component of the SCR that is needed at the end of year 1, we do not apply discounting because all the interest on the risk capital fund must be returned to the capital provider.

Thus our **Solvency Risk Capital** (SCR) at the beginning of year 1 is given by,

$$\mathrm{SCR} = \mathrm{VaR}_{99.5\%}(1) + \sum_{k=2}^{n} \Delta\mathrm{TP}(k) \,.$$

$\mathrm{MVM}(1) = \frac{s*\mathrm{SCR}}{(1+d)}$, but $\mathrm{MVM}(k) = \mathrm{VaR}_{99.5\%}(\mathbf{L}_k)\frac{s}{(1+d)^k}$ based on the unconditional distribution, as before, for $k \geq 2$.



When correlations are taken into account the MVM(1) is augmented so that under a distressed outcome the risk capital can supply the excess needed to bring the TPs up to their conditional values.

Formula (2) above, still holds, but when the correlated nature of the calendar year loss distributions is taken into account the value of MVM(1) is adjusted so that we are prepared for distributional changes that may be required at the beginning of year 2.

If parameter uncertainty is excluded, then there is no correlation between calendar years. In this situation the SCR reverts to the naïve assumption given initially - that is: SCR=VaR(1), and $\mathrm{MVM}(k) = \mathrm{MVM}(k|\xi)$.

Let us consider two scenarios.

<u>Scenario 1: First year is in a distress situation</u>

The amount $\mathrm{VaR}_{99.5\%}(1)$ is exhausted, MVM(1)*(1+d) (plus the risk free rate on the SCR) is returned to the capital providers, and $\Delta\mathrm{TP}(k)$ for $k>2$ is allocated to year k, so that the total provisions are augmented to their conditional values. At the beginning of year two $\mathrm{VaR}_{99.5\%}(2|\xi)$ is raised from the capital providers.

<u>Scenario 2: First year is not in a distress situation</u>

The amount MVM(1)*(1+d) (plus the risk free rate on the SCR) is released to the capital providers plus the minimum of (SCR+Mean Loss - Loss), and SCR.

As illustrated by the above scenarios, the spread above the risk free rate used by the Cost of Capital method should not be confused with the expected excess return over the risk free return the capital providers receives. The expected return on the risk capital must be adjusted by the expected consumption of the risk capital.

Consistent estimates of SCR, Technical Provisions and MVM for one-year risk horizon on updating

In the section on Variation in Estimates of Ultimates conditions for consistency of prior (accident) year ultimates on updating are discussed. It is mentioned that only models and forecast assumptions based on the PTF and MPTF modelling frameworks achieve the necessary conditions of consistency on updating.

Under the same conditions SCR, TP and MVM calculations are also consistent on updating from year to year. For example, if a forecast scenario assumes a calendar trend of 10% +- 2% for next calendar year followed by 5% +- 1% thereafter, then, SCR, TP and MVM are consistent on updating one year hence provided the next year's observed paid losses falls on the assumed 10% +- 2% trend line and the subsequent calendar trend is set to (assumed to be) 5% +- 1%.

Note that for a long tail liability LOB the parameter uncertainty reduces on updating as the model is re-estimated with more data.

Fungibility of Lines of Business and Calendar Years

By default, we assume that the there is no fungibility between calendar years, but LOBs are fungible. That is, a surplus in one LOB can supplement a deficit in another LOB. In practice, this is not necessarily the case for legal or other reasons. Thus, we provide additional options to describe fungibility as follows:

* All lines are fungible, calendar years going forward are fungible (default in ICRFS-Plus™)
* Lines of business are not fungible, calendar years going forward are fungible
* All lines are fungible, calendar years are not fungible
* No fungibility either by Line of Business or by calendar year

The above options are important as they describe the level of diversification afforded for writing multiple Lines of Business - for MVM and Risk Capital. Fungibility applies to the risk capital fund and whether surpluses supplement the risk capital fund (which lowers the required risk capital) or whether the risk fund has to stand on its own.

For more details, see http://www.insureware.com/Library/SolvencyII/solvencyii_ifrs4.php.

Proportionality principle and Standard Formula

Quantitative Impact Studies QIS4 and QIS5 run by CEIOPS postulate the principle of proportionality in the Standard Formula (see for example paragraph SCR.1.3 of Technical specifications for QIS5). This principle assumes VaR is proportional to the BEL for the future years. According to the QIS5, the insurer should assess whether the proportionality principle is applicable.

Based on the research conducted in Insureware, proportionality does not hold for long-tail non-life insurance risk, manifesting the need for the Solvency II Internal model.

Using one year risk horizon metrics module in ICRFS-Plus™, the assertion of inapplicability of the proportionality principle can easily be verified.

One year risk horizon metrics module
ICRFS-Plus™ estimates SCR and MVM for one year risk horizon, based on the identified composite model for multiple LOBs with the associated forecast scenarios.

One year risk horizon metrics module performs the following operations:

- runs the identified composite model with the associated forecasting scenario that predicts lognormal distributions for each cell and their correlations for each LOB;
- based on forecast results, computes BEL as the sum of present value mean losses;
- uses the PALD module to simulate from the predicted lognormals to obtain samples of the aggregates by calendar year for each LOB and the aggregate of all LOBs; Approximately 12 million simulation paths generated;
- computes VaR(k) for each calendar year k = 1 to n, for the aggregate of all LOBs; computes MVM(k) as the present value of spread multiplied by VaR(k);
- selects 10,000 of distress simulation paths ξ that fall into the 99.5% quantile interval for the first future calendar period;
- Based on the set of simulation paths ξ, calculates ΔBEL(k) and VaR($k|\xi$) for each calendar year k = 2 to n;
- computes ΔMVM(k) as spread multiplied by present value of difference between VaR(k) and conditional VaR($k|\xi$) for each calendar year k = 2 to n;
- computes ΔTP(k) as the sum of ΔBEL(k) and ΔMVM(k) for each calendar year k = 2 to n; computes ΔTP as the sum of ΔTP(k);
- calculates SCR as VaR(1) plus ΔTP;
- computes MVM as the sum of MVM(k) plus spread multiplied by present value of ΔTP (MVM(1) is an unadjusted value in this context);
- computes TP as the sum of BEL and MVM

Calculating MVMs using the Cost of Capital method is a doable task provided risk capital allocation by calendar year is available from the predictive distributions for each calendar year (and their correlations). It is possible to customise the approach presented (e.g. integrate Insurer' margin; implement tax adjustments; use Percentiles method instead of Cost of Capital method to calculate MVM etc.). But it is hard to develop a meaningful approach without predictive loss distributions that are produced by ICRFS-Plus™ composite model for multiple LOBs.

References:

Ernst & Young's report "Market Value Margins for Insurance Liabilities in Financial Reporting and Solvency Applications" , 2007. The Ernst & Young's report describes the Cost of Capital method and contains some valuable examples, - Appendix D, Tables 30 and 32, and Appendix E referring to ICRFS-Plus™.